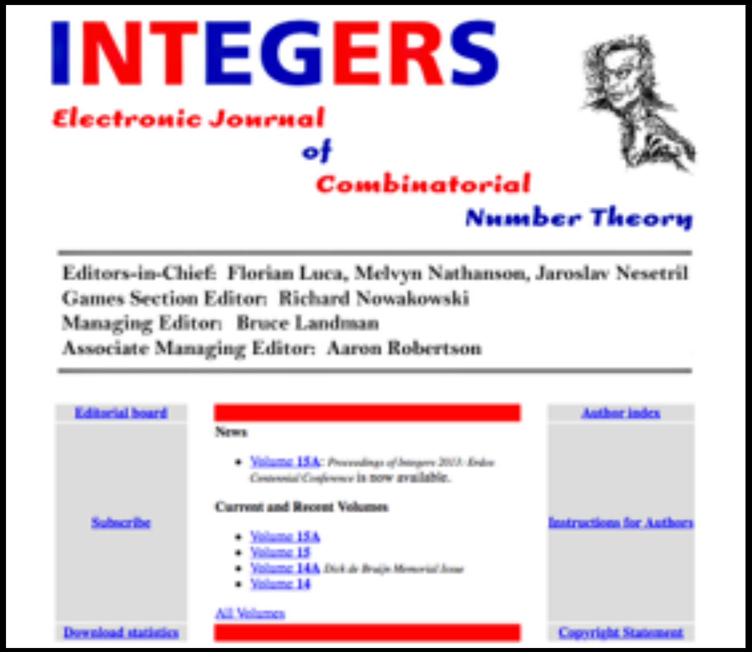
Many people think it impossible for high school students to discover new formulas and theorems, but they are wrong!

Here, we present a very simple method for high school mathematics research project

Our club has produced more than 30 refereed papers (nine countries, and our students talked at international conferences in six countries.



Any one can learn to be creative, but he or she need a good chance to develop talents.

You do not have to be young to cultivate creativity.

We are going to present a method of research with some examples

A Japanese Dice Game
Gambling

Problem 1.

We throw two dices.
Please calculate the probability of the sum of the rolls of these dices is even.

A dice	Odd	Even
probability	1/2	1/2

Two dices A & B	Odd & Even	Even & Odd	Even & Even	Odd & Odd
The sum of two dices	Odd	Odd	Even	Even
probability	1/2 X 1/2 = 1/4	1/2 X 1/2 = 1/4	1/2 X 1/2 = 1/4	1/2 X 1/2 = 1/4

Now we begin to do math research with this problem.
Can you make a new one out of this problem?

Problem 1.
We throw two dices.
Please calculate the probability of the sum of the rolls of these dices is even.

Propose 1.1.

We throw three (or more) dices.
Please calculate the probability of the sum of the rolls of these dices is even.

This is not a new problem. You can find this in a text book.

Propose 1.2.

We throw two dices.

Please calculate the probability of the difference of the rolls of these dices is even.

This is not a new problem. You can find this in a text book.

Propose 1.3.

We throw two dices.

Please calculate the probability of the sum of the rolls of these dices is a multiple of three.

This is not a new problem. You can find this in a text book.

Propose 1.4.

We throw <u>two</u> <u>playing cards</u>.

Please calculate the probability of the sum of the rolls of these <u>playing cards</u> is even.

This is a new problem,
so this is a good problem.

The answer for Propose 1.4.

The probability for the sum to be even is

$$({}_{28}C_2 + {}_{24}C_2) / {}_{52}C_2 = 0.493212$$

The probability for the sum to be odd is

$$({}_{28}C_1 \times {}_{24}C_1) / {}_{52}C_2 = 0.506787$$

number of cards	odd	odd(%)	even	even(%)
2	672	0.506787	654	0.493212

Propose 1.5.

We throw three (or more) playing cards.

Please calculate the probability of the sum of the rolls of these playing cards is even.

The answer for Propose 1.5.

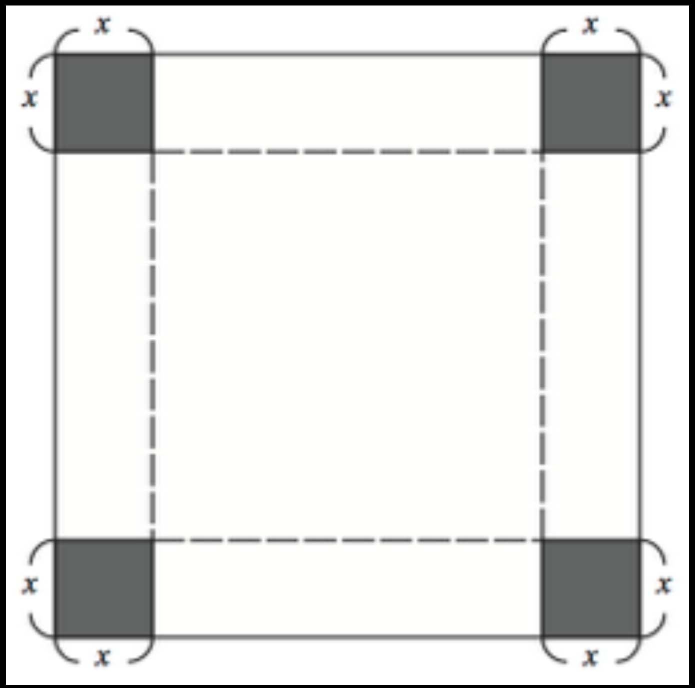
r	Bigger	odd (r)	odd (r) / ${}_{52}C_r$	even (r)	even (r) / ${}_{52}C_r$
1	odd	28	0.5384615385	24	0.4615384615
2	odd	672	0.5067873303	654	0.4932126697
3	even	11 004	0.4979185520	11 096	0.5020814480
4	even	135 296	0.4997543633	135 429	0.5002456367
5	odd	1 299 984	0.5001939237	1 298 976	0.4998060763
6	odd	10 179 456	0.5000096274	10 179 064	0.4999903726
7	even	66 888 784	0.4999738684	66 895 776	0.5000261316
8	odd	376 269 696	0.5000008252	376 268 454	0.4999991748
9	odd	1 839 554 904	0.5000046762	1 839 520 496	0.4999953238
10	even	7 910 002 496	0.4999993923	7 910 021 724	0.5000006077
11	even	30 201 800 664	0.4999989445	30 201 928 176	0.5000010555
12	odd	103 189 758 336	0.5000002660	103 189 648 534	0.4999997340
13	odd	317 506 963 984	0.5000002900	317 506 595 616	0.4999997100
14	even	884 482 962 816	0.4999998816	884 483 381 784	0.5000001184
15	even	2 240 690 280 144	0.4999999056	2 240 691 126 176	0.5000000944
16	odd	5 181 597 854 336	0.5000000582	5 181 596 647 779	0.4999999418
17	odd	10 972 794 957 444	0.5000000355	10 972 793 399 976	0.4999999645
18	even	21 335 987 301 216	0.4999999677	21 335 990 060 434	0.5000000323
19	even	38 180 189 126 884	0.4999999850	38 180 191 415 016	0.5000000150
20	odd	62 997 316 521 216	0.5000000204	62 997 311 372 919	0.4999999796
21	odd	95 995 908 274 464	0.5000000068	95 995 905 659 456	0.4999999932
22	even	135 266 955 835 136	0.4999999853	135 266 963 799 024	0.5000000147
23	even	176 435 163 909 024	0.4999999970	176 435 166 048 576	0.5000000030
24	odd	213 192 496 171 776	0.5000000121	213 192 485 860 324	0.4999999879
25	odd	238 775 590 354 000	0.5000000009	238 775 589 521 952	0.4999999991
26	even	247 959 260 857 728	0.4999999887	247 959 272 090 376	0.5000000113
27	odd	238 775 590 354 000	0.5000000009	238 775 589 521 952	0.4999999991
28	odd	213 192 496 171 776	0.5000000121	213 192 485 860 324	0.4999999879
29	even	176 435 163 909 024	0.4999999970	176 435 166 048 576	0.5000000030
30	even	135 266 955 835 136	0.4999999853	135 266 963 799 024	0.5000000147
31	odd	95 995 908 274 464	0.5000000068	95 995 905 659 456	0.4999999932
32	odd	62 997 316 521 216	0.5000000204	62 997 311 372 919	0.4999999796
33	even	38 180 189 126 884	0.4999999850	38 180 191 415 016	0.5000000150
34	even	21 335 987 301 216	0.4999999677	21 335 990 060 434	0.5000000323
35	odd	10 972 794 957 444	0.5000000355	10 972 793 399 976	0.4999999645
36	odd	5 181 597 854 336	0.5000000582	5 181 596 647 779	0.4999999418
37	even	2 240 690 280 144	0.4999999056	2 240 691 126 176	0.5000000944
38	even	884 482 962 816	0.4999998816	884 483 381 784	0.5000001184
39	odd	317 506 963 984	0.5000002900	317 506 595 616	0.4999997100
40	odd	103 189 758 336	0.5000002660	103 189 648 534	0.4999997340
41	even	30 201 800 664	0.4999989445	30 201 928 176	0.5000010555
42	even	7 910 002 496	0.4999993923	7 910 021 724	0.5000006077
43	odd	1 839 554 904	0.5000046762	1 839 520 496	0.4999953238
44	odd	376 269 696	0.5000008252	376 268 454	0.4999991748
45	even	66 888 784	0.4999738684	66 895 776	0.5000261316
46	odd	10 179 456	0.5000096274	10 179 064	0.4999903726
47	odd	1 299 984	0.5001939237	1 298 976	0.4998060763
48	even	135 296	0.4997543633	135 429	0.5002456367
49	even	11 004	0.4979185520	11 096	0.5020814480
50	odd	672	0.5067873303	654	0.4932126697
51	odd	28	0.5384615385	24	0.4615384615
52	even	0	0	1	1.000000000

There are 28 cases that the probability of getting odd number is bigger than that of even number.

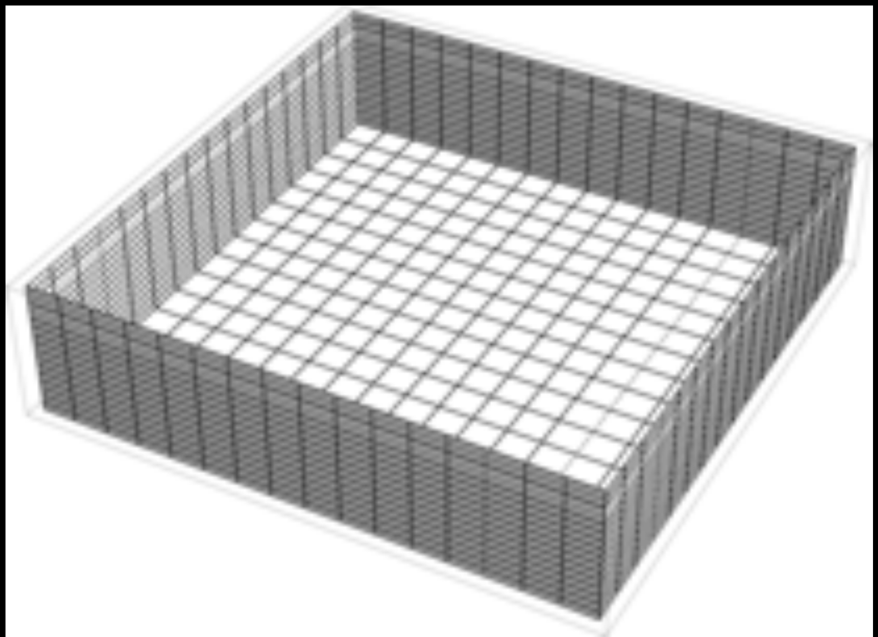
There are 24 cases that the probability of getting even number is bigger than that of odd number.

Problem 2.
Origami Cup Problem.

We cut off squares of x cm $\times$ x cm from the square sheet of paper in Graph 1, and make the rectangular parallelepiped in Graph 2.



Graph 1



Graph 2

Problem 2.
Origami Cup Problem.

We cut off squares of x cm $\times$ x cm
from the square sheet of paper, and make the
rectangular parallelepiped.

Now we begin to do math research with this problem.
Can you make a new one out of this problem?

Problem 2.
We cut off squares of x cm $\times$ x cm from the square sheet of paper, and make the rectangular parallelepiped.

Propose 2.1.

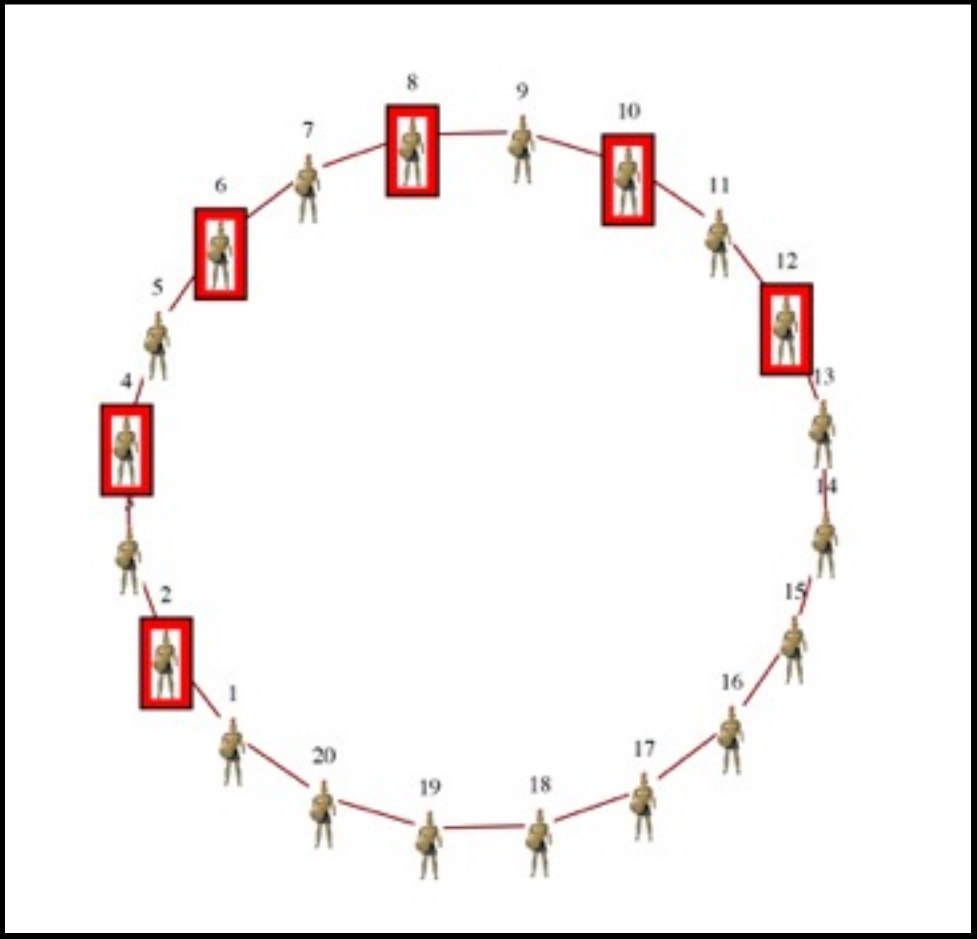
We cut off squares of x cm × x cm from the square sheet of paper, and make the rectangular parallelepiped.

Propose 2.2.

We cut off squares of x cm $\times$ x cm from the square sheet of paper, and make the rectangular parallelepiped.

Problem 3.
Josephus problem

There are people in a circle, and we remove every other man clockwise starting with the first man. The last man will not be removed. Who is the last man?



Let’s make a new problem. First, we change the problem into a simpler one.

Problem 3.
We put numbers in a circle, and remove every other numbers clockwise starting with the number one. Which is the last number?

Propose 3.1.

We put numbers in a circle, and remove every <u>third number</u>clockwise starting with the first number.
Which is the last number?

Propose 3.2.

Put number in a circle. There are two process of removing every other numbers. <u>One process start with the first number, and it goes clockwise. Another process starts with the last number, and goes counterclockwise.</u>

Propose 3.3.

Put Numbers in a line We starts with the first number, and remove every other number.

Once we reach the other end, we change the direction.

Which is the last number?