
Grundy Number

Mathematical Sciences Club

What is Grundy number?

Grundy number is defined by the following definitions.

The definition of move-function

The function “move” present all the positions that can be reached from the present position in an option.

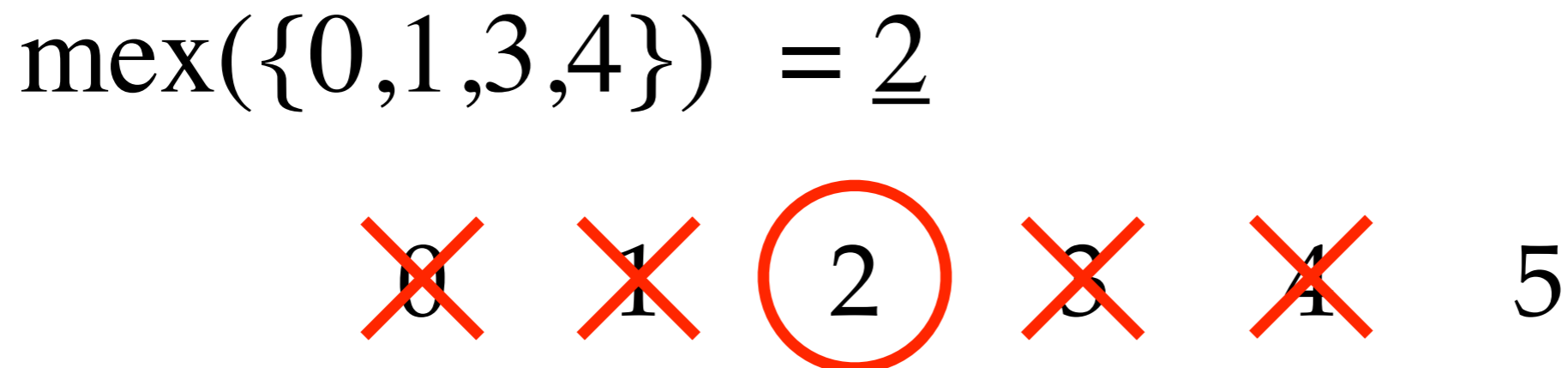
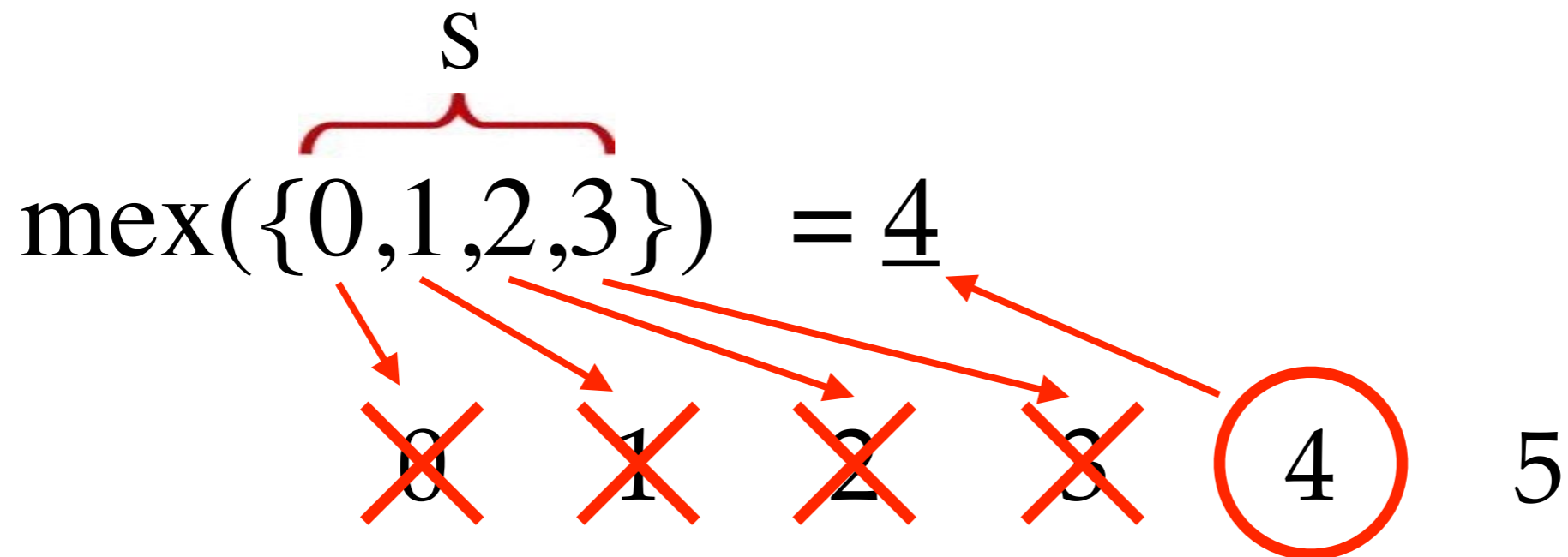
The definition of mex-function

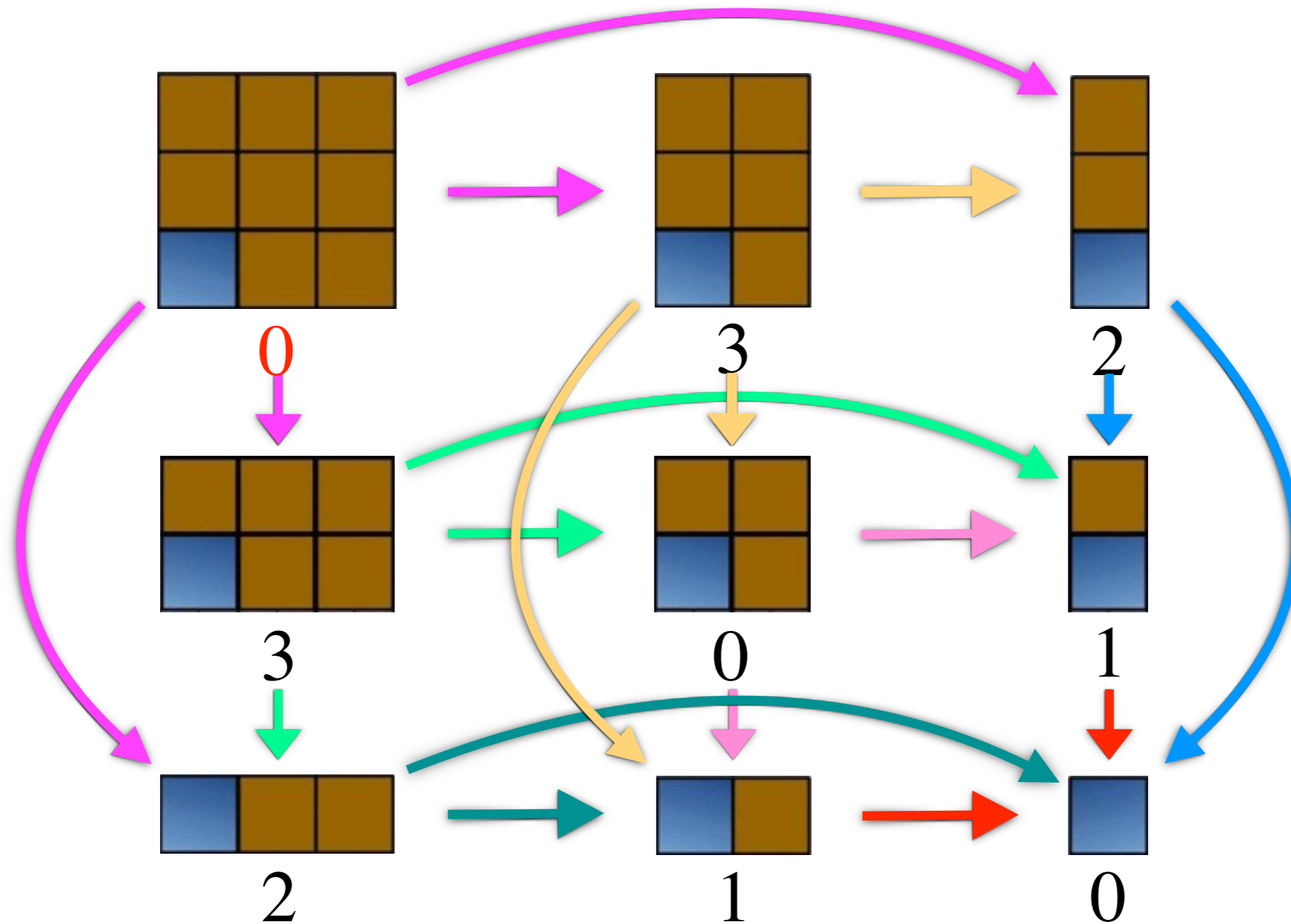
$\text{mex}(S)$ is the least non-negative integer that is not included in S .

An example of “mex”.

mex-function

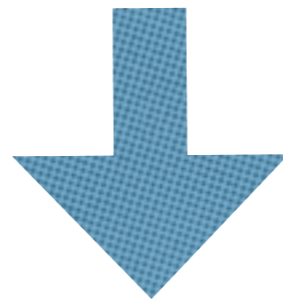
mex(S) is the least non-negative integer that is not included in S.





- From a position of Grundy number 0,
we always move to a position of positive Grundy number.
- When I start with a position of positive Grundy number,
I can move to a position of Grundy number 0.

- From a position of Grundy number 0, we always move to a position of positive Grundy number.
- When I start with a position of positive Grundy number, I can move to a position of Grundy number 0.



- If I start with a position with Grundy number 0, my opponent can win the game using the optimal strategy.
- If I start with a position with positive Grundy number, I can win the game using the optimal strategy.

The definition of Grundy number

The definition of move-function

The function “move” present all the positions that can be reached from the present position in an option.

The definition of mex-function

$\text{mex}(S)$ is the least non-negative integer that is not included in S .

$$G(y, z) = \text{mex}\{G(\{u, v\}); \{u, v\} \in \text{move}(\{y, z\})\}$$

In summary,

- If I start with a position with Grundy number 0,
my opponent can win the game using the optimal strategy.
➡ Next player's position
- If I start with a position with positive Grundy number,
I can win the game using the optimal strategy.
➡ Previous player's position

In chocolate games, there are two outcome classes.

- N-Position ➡ Next player's position
 - P-Position ➡ Previous player's position
- ➡ When Grundy number is 0, we have P-Position